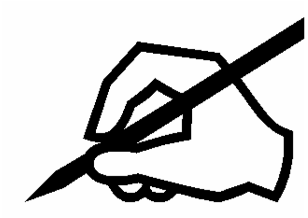
National Open School of Trinidad and Tobago



LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT ONE:
NUMBER THEORY

Lesson 1

Understanding the Number System

Textbook: Mathematics, A complete course, Volume One.

(Some helpful exercises and page numbers are given throughout the lesson, e.g. Ex 2f page 27)

INTRODUCTION

.
This lesson introduces sets of numbers and some of the basic properties that characterise members of these sets. This topic is extremely important because most, if not all, of the mathematical operations involve manipulating members of these sets. The properties allow for certain calculations and simplifications of mathematical expressions which you will learn as the course progresses.

OBJECTIVES

At the end of this lesson you will be able to:

a) Distinguish between sets of numbers;
b) Identify a set of numbers as a subset of another set;
c) Describe integers as being prime or composite;
d) Compute the H.C.F or L.C.M of integers;
e) Use number properties to simplify computational task
f) State the place value of a digit in any base.

1.1 Sets of Numbers

A detailed description of sets of numbers is given in your textbook, pages (14–16). The following are some of the more important teaching points. The symbols that represent the different sets are given in brackets.

- Natural numbers (N) are the counting numbers e.g. 1, 2, 3, etc.

- Whole numbers (W) are natural numbers and zero. NB. Zero is not a natural number.

- Integers (Z) are negative and positive whole numbers and zero e.g. -2, 0, 5 etc.

- Rational numbers (Q) are negative and positive fractions. These include all whole numbers since they can be written as fractions with a denominator of 1,

  e.g $7 = \frac{7}{1}$, $0 = \frac{0}{1}$.

- Irrational numbers (Q') cannot be written as fractions e.g. π and $\sqrt{3}$. The expressions 3.14

  and $\frac{22}{7}$ are approximations of π . Decimals that do not recur or do not terminate are also irrational. NB. Recurring decimals are rational.

- Real numbers (R) are rational and irrational numbers. It includes all the sets mentioned above.

- The set of natural numbers is a subset of the set of whole numbers. Simply put natural numbers are whole numbers. Also, whole numbers are integers and integers are rational numbers. This can be represented by $N \subset W \subset Z \subset Q \subset R$.

ACTIVITY 1

Choose the letter symbol that best represents the set that is described by each of the following.

(i) This set has a zero as a member. N or W

(ii) This set contains decimals that do not terminate or recur. Q or Q'
(iii) This set has negative and positive fractions. Z or Q

1.2 Number Properties

The following laws or properties are important. See textbook, pages (17 – 18)

- Identity
  The identity for addition is 0 and the identity for multiplication is 1. When a number combines with the identity under an operation it remains unchanged.

  Example: 5 + 0 = 5 and 7 x 1 = 7.

- Inverse
  If a number combines with its inverse under an operation the result is the identity. Each number has its own inverse.

  Example: (i) The inverse of 8 under addition is −8.   Therefore 8 + (−8) = 0

  (ii) The inverse for 8 under multiplication is $\frac{1}{8}$. Therefore 8 $\times$ $\frac{1}{8}$ = 1

- Law of closure
  If when any two members of a set are combined under an operation the result is always a member of the set then the set is closed under the operation. Whole numbers are closed under addition e.g. 6 + 3 = 9. It is not close under subtraction since an integer can result from subtracting two whole numbers e.g. 7 – 9 = −2. Can you think of any set of numbers that may be closed under subtraction?

- Commutative Law
  Addition and multiplication of numbers are commutative. Therefore the order in which the operations are performed do not matter, the results are the same.

  Example: 45 +23 = 68 and 23 +45 = 68.

  Are subtraction and division commutative?

- Associative Law
  Addition and multiplication are associative. Therefore numbers can be grouped differently to add or to multiply the results are the same.

Example: $(3 \times 5) \times 6 = 3 \times (5 \times 6)$

$15 \times 6 = 3 \times 30$

- Distributive Law
  You can use the distributive law to simplify computations that involve multiplication over addition or subtraction.

Example: $7(4 + 8) = 7 \times 4 + 7 \times 8$
$= 28 + 56$
$= 84.$

ACTIVITY 2

Simplify the following using the distributive law.

(i) $5(6 – 4)$

(ii) $6(9 + 8)$

(iii) $3(9 + 6 – 2)$

1.3 Concept of H.C.F and L.C.M

Let us review a few basic concepts.

- Factors of a number can divide it without leaving a remainder. 1 is a factor of any number. Any number is a factor of itself. The factors of 6, are 1, 2, 3, and 6.

- Prime numbers only have two factors 1 and itself, e.g. 2, 3, 5 and 7 are all prime numbers. NB. 1 is not a prime number and 2 is the only even prime number.

- Composite numbers have more than two factors, e.g. 4, 9, and 12 are composite numbers. For example, factors of 4 are 1, 2, and 4.

- Multiples of a number can be obtained by counting in groups of the number, e.g. multiples of 2 are 2, 4, 6, 8, 10, etc

Highest Common Factor (H.C.F.)

H.C.F of two or more positive integers can be found by listing all the factors of each integer and the choosing the greatest factor common to all.

Example: Find the H.C.F of the numbers 12, 18 and 24.

Factors of 12 = 1, 2, 3, 4, 6, 12

Factors of 18 = 1, 2, 3, 6, 9, 18

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

1, 2, 3, and 6 are common factors but clearly 6 is the H.C.F.

Alternative Method (repeated division)

This involves dividing repeatedly by factors of all the numbers and then multiplying the factors. It is best to divide by prime numbers.

$$\begin{array}{r|l} 2 & 12,\ 18,\ 24 \\ \hline 3 & 6,\ \ 9,\ 12 \\ \hline & 2,\ \ 3,\ \ 4 \end{array}$$

There are no more common factors to divide by other than 1 therefore the H.C.F is 2 x 3 = 6.

Lowest Common Multiple (L.C.M)

To determine the L.C.M of two or more positive integer you can list multiples of each integer and choose the smallest common multiple. However this can become very cumbersome. The best method is repeated division. Like H.C.F it is best to divide by prime numbers but unlike H.C.F a number is used to divide once it is a factor of any one of the integer.

Example: Find the L.C.M of 2, 6 and 9.

$$\begin{array}{r|lll} 2 & 2, & 6, & 9 \\ \hline 3 & 1, & 3, & 9 \longleftarrow \text{2 cannot divide 9 exactly, therefore bring it down.} \\ \hline 3 & 1, & 1, & 3 \\ \hline & 1, & 1, & 1 \end{array}$$

The L.C.M is $2 \times 3 \times 3 = 18$.

H.C.F and L.C.M may be used to solve real life problems. Words such as largest and greatest may indicate that H.C.F can be used to solve. While words such as lowest, least, and smallest indicate that L.C.M may be used.

Example: A room measures 450 cm by 250 cm. Determine the length of the largest square tile that can be used to tile the floor without cutting.

Determine the H.C.F.

2	250,	450
5	125,	225
5	25,	45
	5,	9

The length of the largest square tile is $2 \times 5 \times 5 = 50$ cm

ACTIVITY 3

1. In a school, it is possible to divide the pupils into equal sized classes of either 24 or 30 or 36 pupils and have no pupils left over. Determine the least number of pupils that can make this possible. How many classes will there be if each class is to have 30 pupils? (Ex 2f, page27)

2. What is the smallest number of sweets that can be shared exactly among 5, 10 or 15 students?

1.4 Place Value and Number Bases

You must be able to state the place value of a digit in any base. We count in groups of ten (base ten) therefore each digit has a place value in terms of powers of ten.

Example: State the value of the 5 and 2 in the number 6590.42. Consider the following table.

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}
6	5	9	0	4	2

The powers after the decimal point are negative.

The value of the 5 is $5 \times 10^2 = 5 \times 100$ or 500.

The value of the 2 is $2 \times 10^{-2} = 2 \times \frac{1}{100}$ or $\frac{2}{100} = \frac{1}{50}$.

What if the base is 2?

Determine the value of the first1 in 101_2

Consider the following table:

2^2	2^1	2^0
1	0	1

Solution: $1 \times 2^2 = 1 \times 4$ or 4

ACTIVITY 4

Determine the value of the 4 in:

(i) 423_5
(ii) 5.674_8

ASSESSMENT

1. Which of the following is not a rational number?

(A) 0

(B) 2.5

(C) π

(D) $\frac{1}{3}$

2. Which of the following is the most suitable set that a teacher may use to state the heights of students in her class?

(A) Natural numbers

(B) Whole numbers

(C) Integers

(D) Rational numbers

CONCLUSION

We have looked at different sets of numbers and properties of these sets. Remember that these properties are used to simplify computations and solve equations. In the next lesson we will look at a number of computational tasks that involve manipulating real numbers.